

POSIBILIDADES Y LIMITACIONES PARA UN ENCAJE INTERDISCIPLINAR DEL CURRÍCULO DE MATEMÁTICAS

POSSIBILITIES AND LIMITATIONS FOR AN INTERDISCIPLINARY APPROACH OF MATHEMATICS CURRICULUM

Álvaro Sánchez González.

PhD in Mathematics. Secondary School Teacher in the Community of Madrid. Associate Professor in the Faculty of Mathematics at the Complutense University of Madrid.

Resumen

Las Matemáticas se han señalado como una disciplina fundamental en la formación del alumnado, así como de cara a la adquisición de las competencias necesarias para los individuos de la sociedad del siglo XXI. Por otro lado, el contexto social junto al paradigma educativo actual está tendiendo a una visión integral del conocimiento, que se materializa en la interdisciplinariedad como forma de estudiar un fenómeno de la realidad.

Analizamos las posibilidades y dificultades que la coyuntura educativa nos plantea para trasladar esta visión interdisciplinar a las aulas. Realizamos una revisión histórica de ejemplos de interrelación de las Matemáticas con otras disciplinas y aportamos sugerencias para una implementación del currículo de Matemáticas con un punto de vista interdisciplinar.

Palabras clave: *Educación, Interdisciplinariedad, Matemáticas, Métodos pedagógicos.*

Abstract

Mathematics has been pointed out as a fundamental discipline in the formation of the students as well as in the acquisition of the necessary competences for the individuals of the society of the XXI century. On the other hand, the social context together with the current educational paradigm is tending towards an integral vision of knowledge, which is materialized in interdisciplinarity as a way of studying a phenomenon of reality.

We analyze the possibilities and difficulties that the current educational situation poses to transfer this interdisciplinary vision to the classroom. We make a historical review of examples of interrelation of Mathematics with other disciplines and provide suggestions for an implementation of the Mathematics curriculum with an interdisciplinary point of view.

Keywords: *Education, Interdisciplinarity, Mathematics, Pedagogical methods.*

1. THE COMPETENCY AND INTERDISCIPLINARY FRAMEWORK IN EDUCATION

The access to education established in the Universal Declaration of Human Rights in the mid-twentieth century, and ratified in the Spanish Constitution of 1978, placed in the hands of educational centers the training of future adult citizens. Since then, profound changes have taken place in society, to which education is no stranger and, undoubtedly, it must respond to the demands that arise as a result of the complex reality in which we live.

The current social paradigm makes it essential for each person to master a series of resources and, in addition, to have the ability to apply them. Here we have a first approach to what we call being competent: each individual is required to know and efficiently use techniques to get by on a daily basis, as well as in his or her work environment. And not only that, but the dizzying technological evolution we have been experiencing since the middle of the last century means that knowledge soon becomes obsolete and needs to be updated, so the individual is also expected to have the ability to incorporate new knowledge throughout his or her life.

In short, it is the description of the profile that society ideally demanded of an adult citizen at the dawn of the 21st century. Accordingly, the United Nations Organization [UN], through its specialized agency for Education, Science and Culture [UNESCO], published the well-known Delors Report (1996) as the leading text to guide educational plans, at least in developed countries. In that publication, the Competencies were established as the reference around which the teaching-learning process should revolve and which, in Spain, were included in the Organic Law on Education 2/2006 [LOE], recently amended by Organic Law 3/2020 [LOMLOE]. However, more than a quarter of a century has passed

since the publication of the Delors Report and it has become necessary to update the objectives of education in order to respond to new social demands.

On the other hand, every individual is constantly faced with decisions that should be made after interpreting the information available, as well as according to various criteria of personal or, sometimes, corporate perception. The globalization of the economy and social relations, enhanced by Internet browsing, as well as the dilemmas and debates generated on tolerance, environmentalism, sustainability, or health crises, mean that both information and criteria can be given, influenced, or biased by external agents. But it is the individual, ultimately, who must analyze and assume them from a critical perspective to form his or her own opinion on which to base each decision.

The UN identified the challenges that as a society had to face in the coming years and, in 2015, launched the 2030 Agenda in which it promulgated the Sustainable Development Goals. Subsequently, the Organisation for Economic Co-operation and Development [OECD] established the Learning Framework 2030 - Compass 2030 for Education in which it is recognized that "the concept of competence implies more than the mere acquisition of knowledge and skills [...] both broad and specialized knowledge will be required". And it continues: "Knowledge of a discipline will continue to be necessary [...] along with the ability to think across disciplinary boundaries and connect them".

It is in this situation of constant change, in which new challenges and objectives are enacted at the global level, that UNESCO commissions the elaboration of a new report entitled *Reimagining our futures together: a new social contract for education* (Sahle-Work Zewde, 2021) and whose purpose is simply to take up the baton from the Delors Report, The new report repeatedly insists on the need to apply the principles of cooperation and collaboration. Moreover, it explicitly states as a proposal for improving education that "curricula should

emphasize ecological, intercultural and interdisciplinary learning that helps learners to access and produce knowledge, while developing their capacity to critique and apply it".

We will take the above suggestion as a point of reference in this article and will focus on the possibility of developing interdisciplinary experiences organized from or around Mathematics to contribute to the achievement of an adequate competency profile of students. First of all, we will make some clarifications about interdisciplinarity and we will analyze the conditioning factors, both for and against, that the Educational System imposes on us in order to transfer it to the classroom. We will recover some examples in which this approach has been fruitful for Mathematics for the development of Knowledge and, finally, we will make some observations for the implementation of an interdisciplinary model from the perspective of the Mathematics curriculum.

2. INTERDISCIPLINARITY: METHODOLOGY OR ATTITUDE?

Since ancient times there has been a gradual process of segmentation in the study of the phenomena that surround us, thus creating the different disciplines of knowledge. The educational plans of the twentieth century were designed following this tradition of fragmentation (Torres, 1994, chapter one): as the age of the students advances, the curricula are subdivided into subjects with watertight and self-contained programs, which are analogous to different plots of knowledge, as well as to specialization in production processes.

As García and de Alba (2008) point out, it seems evident that the response to global challenges and complex phenomena cannot be achieved from the current fragmented approaches to knowledge in a multitude of academic disciplines. To cite a few examples, it is inconceivable to study a biological process without considering the underlying chemistry, nor is the study of Physics

understood without understanding mathematical language. The same is true of the Social Sciences in general, or the Humanities and the Arts, such as History or Music, which interact with each other almost spontaneously and constantly.

In order to deal with partial approaches, discourses that seek to integrate the different tasks of each discipline have arisen. This leads to the so-called integrated curriculum, well founded and developed in the book by Torres (1994). This approach challenges the traditional division into watertight subjects (Valdés, 2017) and undoubtedly advocates academic interdisciplinarity as a strategy to materialize a global vision in the classroom.

We warn here about the nature of the term *interdiscipline*, since it is sometimes attributed nuances that differentiate it from others such as *multidiscipline* and *transdiscipline*, not always well used as synonyms. Beyond the subtle differences between the two terms, we take for granted Piaget's (1978) classic definition, which requires that between disciplines there be "reciprocity of exchanges that result in mutual enrichment". Thus, interdiscipline implies the recognition of the inadequacy of the tools of each discipline alone. It also requires a collaboration of the professionals of the disciplines, not remaining in the mere aggregation of the contributions of each discipline, but elevating and giving a higher level knowledge that would have been impossible separately. In the words of Welch (2011) "interdisciplinary practice has to transcend the disciplinary structures of knowledge". And, further on, he concludes that interdisciplinarity does not seek the destruction of disciplines, but rather seeks to make use of them "by expanding their contexts and establishing synthetic relationships between them".

However, in the pursuit of interdisciplinarity, the fact that each discipline works with its own methods can become a point of conflict, and although in a generalist framework we would be able to recognize a Science as rigorous, the

standards of rigor are different between each of them (Follari, 2007). In any case, if interdisciplinarity is natural in the professional world, there is apparently no reason not to resort to it in order to enrich the teaching-learning process.

Indeed, the fact of blurring the boundaries that compartmentalize knowledge is one of the positive aspects that make it advisable to use interdisciplinarity, as cited by Fiallo (2001). Similarly, this author cites other benefits to be taken into account, such as the development of group work skills and increased student motivation, as well as avoiding repetitions in the curricula of different subjects, which helps to optimize the time of teachers and students in the classroom. Furthermore, Fiallo suggests the relevance of articulating interdisciplinarity around Mathematics since "the concepts, procedures and attitudes that mathematics and the rest of the disciplines can be associated to main nodes that are distinguished by their applications to social practice".

Figure 1. *Interdisciplinarity advocates the creation of knowledge from the integrative vision of several disciplines..*

Interdisciplinarity advocates the creation of knowledge from the integrative vision of several disciplines.

For an interdisciplinary approach to be possible, there must first be a work of analysis from each discipline, and then an integrative synthesis back to the global vision. It is worth insisting that the interdisciplinary approach does not seek to put an end to specialization, nor does it go against it, but rather seeks to use it to raise the search for knowledge to a higher level, thus overcoming fragmentation (López, 2012). It is undeniable that there are problems and objects of study in research that cannot be treated from a single perspective, therefore, interdisciplinarity is but the necessary consequence of the insatiable search for

knowledge that seeks to be done in an integral way.

In short, when transferring the previous discourse aspiring to achieve a certain degree of interdisciplinarity in the classroom, we should not consider this a methodology in itself but rather a methodological attitude that seeks interaction between the different subjects of the curriculum. However, given that the curricula are legislatively organized in subjects with established programs that must be respected, an analysis of the educational system must be carried out in order to detect the margin of action that exists for the realization of a possible interdisciplinary proposal.

3. EDUCATIONAL SYSTEM CONSTRAINTS

If we intend to establish connections with the real world from the classroom, then we are obliged to present the objects of study from all possible perspectives. As we have already argued, the study of any phenomenon from a single discipline will only provide a partial view of it, whereas, as we pointed out earlier, interdisciplinarity seems to be the natural approach in much of the scientific world. Consequently, in educational approaches it seems appropriate to resort to organizational strategies that favor interdisciplinarity or, at least, multidisciplinary.

Before putting forward any proposal, we must make a study of the opportunities offered by the educational system, as well as the limitations that exist within it. We resort to the typical SWOT (Strengths, Weaknesses, Opportunities, and Threats) analysis scheme, grouping together the beneficial aspects on the one hand, and those that could work against us on the other.

3.1. OPORTUNITIES AND STRENGTHS

The Spanish education system has been changing since the restoration of

democracy. The first democratic governments inherited the General Education Law of 1970, which had already meant a certain openness to avant-garde pedagogical currents (Gozzer, 1982). During the following fifty years, a series of attempts have been made to modernize and adapt the system to social requirements, as well as to recommendations in line with teaching theories.

The legislation enacted in 1990 and 2006, namely: the Organic Law for the General Organization of the Educational System [LOGSE] and the Organic Law on Education [LOE], stand out from the rest because of their major structural changes. The LOGSE was intended to be a major reform that sought to improve the quality of education and, in fact, according to the statements of the Director General of Secondary Education at the time, as reported by Merchán Iglesias (2021), the intention was to “encourage new methodologies, dynamics and classroom structures” in what came to be called the Pedagogical Reform, that is, a profound reform of the curricula. However, most of those innovative proposals were not successful and were hardly put into practice.

Already at the beginning of the 21st century, the entry into force of the LOE caused the assumption of supranational educational ideas coming from the EU and UNESCO, highlighting among these precepts that of an education focused on the competency model. In fact, in its last modification through the LOMLOE, the influences of Agenda 2030 and the SDGs, mentioned above, are explicitly recognized in the preamble. Moreover, the LOMLOE for the first time explicitly incorporates interdisciplinarity as a possibility within the offer of electives in the Secondary stage, which we can consider an opportunity to develop such an approach in the centers.

However, whatever legislation is enacted, it is the teachers who implement it. As can be inferred from studies such as those by Pérez-Díaz and Rodríguez (2013) or Gómez (2020), the teaching staff always raises social debate about its

prestige and suitability, although the truth is that these studies show that one of the strengths of the educational system lies precisely in its commitment to students and their families. The training of teachers both in their subject and in general knowledge is also positively valued, as well as the progress made in recent years in the use of technological resources in the classroom (Valdés et al., 2021). The multidisciplinary and interdisciplinary experiences that are carried out each academic year in many centers are varied, many of them remaining anonymous, but others being published and even awarded in different types of calls. We endorse the final reflection of López and Gustems (2007) in which they argue that "institutions that have teams of teachers and researchers from different backgrounds should make the most of this potential for the future".

Examining the avenues that the legislation leaves open to implement interdisciplinary experiences, we note in the first instance that there is the possibility for schools to adopt an organization by areas, although for the time being the majority rule continues to be that of a traditional distribution of compartmentalized subjects. In addition to this legislated option, interdisciplinarity is also making headway encouraged through institutional programs such as eTwinning of the Ministry of Education, Simbiontes of the Polytechnic University of Madrid, or the agreements reached in 2013 to develop in certain Centers the curricula of the International Baccalaureate Organization and that, in fact, the State School Council now recommends to promote through scholarship programs in the Report on the State of Education in Spain 2022.

Recalling that true interdisciplinarity requires establishing interrelationships between the contents, methods and other didactic components of different subjects, the Centers must make effective use of the autonomy attributed to them by the Legislation and stipulate a line of work in which the traditional organization is deconstructed. For the fulfillment of this crucial requirement,

Llano Arana et al., (2016) identify the need to outline concrete and staggered actions in each of the years and from all the subjects of the curriculum, thus constituting a way of traversing the curriculum vertically and horizontally.

In this sense, the role of the Education Inspection Service is also relevant and surely differentiating in order to successfully establish an interdisciplinary routine. Educational innovation is one of the principles that should guide the teaching work, and one of the fundamental points to achieve it is to tend to ensure that students can establish an interdisciplinary vision of the contents that may be prone to it, which can be encouraged not only from the approach in the presentation of the contents by the teacher but "raising activities that allow the treatment of contents whose references are various areas in an interdisciplinary way and with application of key competences" (Vázquez Cano, 2018).

In their role as advisors, inspectors can contribute with their ideas to facilitate the approach of all these vertices and, if necessary, allow an adequate legal fit of the project pursued (Estefanía Lera, 2017). It would be desirable to apply some flexibility to facilitate the complex task of developing a center project focused on interdisciplinarity, which ultimately has as its ultimate goal the learning of students, which only goes in the direction of improving the quality of education (Sáenz, 2022). It almost always requires many hours of coordinated work, mainly on the part of teachers: to devise learning situations and work material, interweaving the agendas of different subjects, methodological changes, training in disciplines other than the one one one masters, teaching or even co-teaching in related subjects, etc. and all this work does not end up culminating without the will of the Inspection Service which, ultimately, must give its approval to the program.

Finally, as regards Mathematics in particular, reference should be made to the exhaustive report of the chapter dedicated to the teaching of the discipline

during the compulsory stages and the Baccalaureate in the White Paper on Mathematics (López Beltrán et al., 2020). We can consider a strength the fact that a series of values and attitudes in relation to Mathematics appear in the curriculum, introduced by the now repealed Organic Law 8/2013, of December 9, for the improvement of educational quality [LOMCE] as a block of transversal content and reformulated in the LOMLOE as part of its specific competencies.

In the aforementioned chapter of the *Libro blanco*, the effort made from the Administration and even the Centers and their Departments to gradually replace the traditional model segmented into disciplines for another in which curricular integration prevails, singularly with the so-called STEM or STEAM (Science, Technology, Engineering, Art and Mathematics) projects, is also pointed out as a strength. In this sense, the amount of digital resources available today is also an undeniable opportunity to articulate interdisciplinarity by making use of them.

As we pointed out in the first part of the article, in order to have interdisciplinarity, the disciplines themselves must first exist. Therefore, as also mentioned in the White Paper, these experiences of curricular integration should be conceived as an opportunity in two senses, “both to understand what and how certain phenomena affect the different mathematical disciplines and give the curriculum a multidisciplinary character, and to recognize the problems that have historically allowed the different disciplines to evolve”. And this is precisely how we will proceed in the next section, once we have explored the aspects that can be a hindrance to interdisciplinary aspirations.

3.2. WEAKNESSES AND THREATS

When the debate on interdisciplinarity arises, we immediately resort to archetypal examples of interaction between disciplines and, almost always, the

examples are limited to those of a scientific nature. However, we often forget that the division between disciplines is mostly a matter of affinity, but does not make them mutually exclusive. Unfortunately, the debate of the two cultures, the misnamed division between humanities and sciences, is still present and deeply rooted.

We do not hesitate to consider uneducated the person who does not know Shakespeare or Cervantes, while ignorance of Euclid or Archimedes is often excused, despite the fact that the former are as relevant to Literature as the latter are to Mathematics. It is easy to see that clichés continue to survive in order to denigrate some and justify the ignorance of others. This debate is sterile, and the proliferation of the Social Sciences bears witness to this, and it is urgent to dismantle this threat by showing the fruitful relationship between disciplines of very different natures.

On the other hand, if in the previous section we discussed the favorable points of the current legislation, it should be pointed out that the enormous and changing number of regulations that have been enacted in barely half a century makes it difficult for any pedagogical proposal to take root sufficiently (Novella and Cloquell, 2022). Moreover, the climate of provisionality generated among teachers, together with the changing composition of the Senate itself, may lead them to think that their efforts will be useless (Campos and Zúñiga, 2020). Once again, the need for a broad agreement among political forces, based on consensus and prior commitment among the educational community, which guarantees the stability of a regulation in such a way that the teaching staff is fearlessly involved in its application, is once again highlighted.

In the meantime, we must make do with the experiences of individual teachers and schools. And we will limit ourselves to pointing out once again the Educational Inspection Service as an ally of these actions, since it would be

wrong to feel it as a threat if it is more inclined to the scrupulous application of the norm than to the advisory work that we have pointed out a few paragraphs above.

Another weakness detected in relation to the legislation is not only the excessive division of the curriculum into disciplines, but also within the subjects themselves. In the corresponding report included in the *Libro blanco de las Matemáticas*, the internal disconnection of the Mathematics curriculum is pointed out, with no links between arithmetic, algebra or statistics, to give some examples. At several levels, the curriculum has reduced the vision of mathematics to a simple conglomerate of unconnected and decontextualized algorithms, stripping the subject of its most important characteristics: logical thinking, rigor and universality.

The *Libro blanco* also identified as weaknesses, among others, the repetition of content, the excessive tendency to memorize processes and routines, as well as an excessive use of inappropriate terminology for each age group in question. The need to introduce a change of vision in the teaching of pre-algebra or geometry is pointed out, in order to create a gradual approach to the maturation of these processes, something that seems not to have been contemplated legislatively for the time being. As for terminology, Kline's criticism (1973/1976) of those who wanted to make their students emulate the neat rigor of the professional mathematician is well known. Of course, this rigor must be preserved as a hallmark of mathematics, and it is the teacher's responsibility to combine it with a register accessible to his pupils.

Possibly, the greatest threat to the training of students in mathematics is, as ironic as it may seem, the teacher's own training in mathematics, something already detected by Guzmán (2007), especially with regard to the integration of knowledge about the cultural repercussions of the knowledge itself. According to

the reports on which the *Libro blanco* is based, there is a lack of training among Primary Education teachers, while the teaching option as a Secondary Education teacher is less and less chosen by university graduates specialized in Mathematics.

This situation makes it difficult to articulate a flexible programming around mathematics because, as we have seen, one of the requirements for this is a broad and deep teacher training both in the specialty and in other related areas and general culture. In addition, the system of access to the teaching career requires specialization in initial teacher training, but at no time is other related training required beyond that which the teacher himself wishes to undertake voluntarily.

According to the conclusions of the experiment carried out by Pozuelos Miranda et al. (2012) "an interdisciplinary experience and its practical development needs to be clearly and precisely presented to a student body that does not always have the same perspective or the same level of knowledge as the teaching team that promotes it". We add here that for this to happen, the teaching team must first have excellent training in various fields. In this regard, it is a weakness that the regulations require a single teacher to teach a whole area that brings together several disciplines and does not facilitate co-teaching by two professionals who complement each other in the classroom.

In order to be in a position to develop an interdisciplinary experience, both the initial training of teachers and their updating throughout their professional careers are key points. These and other issues have been pointed out in several studies. We can again cite Fiallo (2001) who stresses that "subject curricula are eminently disciplinary [...] teacher training must break a training paradigm and interact with other knowledge in which they are not specialists". Precisely, as a consequence of this disciplinary training, everyone considers their discipline to be

the most important in the curriculum, but, on the contrary, we should exercise modesty and place our subjects at the service of others as well as of our own learning.

The last weakness to note, although it can be remedied with will and time, is our own inexperience in interdisciplinary work. We have already mentioned the training of teachers and the much needed time to develop the proposals, which should start from the most modest and punctual to grow based on successes and mistakes until culminating in longitudinal programs that could cover entire courses and several or even all the subjects of the curriculum.

4. MATHEMATICS, THE BACKBONE OF HUMAN KNOWLEDGE

Although Mathematics is conceived as an eminently theoretical and abstract Science in excess, this discipline has never forgotten to cultivate its applied side. Since the first philosophers and mathematicians of Ancient Greece, mathematics has been used to study everyday phenomena and its development in recent centuries has made mathematics the basis of modern science, as well as the development of the industrial, technological and computational world (Quirós and Vázquez, 2006).

The truth is that mathematics in general has proved to be extremely efficient when it comes to being used as a tool to describe phenomena of many different kinds. Not in vain did Galileo say that it was "the language in which Nature is written" and, going even further, Wigner (1960) used the expression "the unreasonable effectiveness of mathematics" to refer to the innumerable examples throughout history that show us that this discipline is capable of providing rigor, predicting phenomena and serving as a theoretical justification for other experimental disciplines.

In the following paragraphs we present a selection of examples that, at different moments in history, illustrate the success of interdisciplinary approaches for the individuality of each of the overlapping disciplines. More detailed versions can be found in almost any book on the History of Mathematics; here we have drawn mainly from Boyer (1968/1986). The choice of these examples is not entirely accidental, since they have clear connections with the Mathematics curriculum and therefore, with the corresponding caveats, can be used to make classroom proposals around them.

4.1. THEORY OF PROPORTIONS AND MUSIC

In Western music today, the use of the just temperament scale introduced by the Germanic J. S. Bach (1685-1750) in his work *El clave bien temperado*, crucial in the development of Baroque music theory, predominates. But the origin of the first fundamentals of the musical scale can be traced back to the Greek civilization of the 6th century B.C. It is said that Pythagoras himself had noticed that by plucking a taut string twice the size of another, both emitted a similar sound that was properly coupled and pleasing to the human ear.

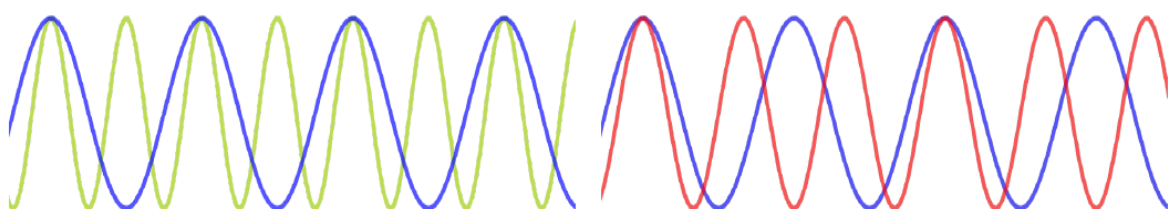
The Pythagoreans, under their belief that behind any phenomenon of the sensible world lay the idea of number, developed a Theory of Proportions which, with some later adjustments by Eudoxus of Cnidus in the 4th century B.C., is practically the one we still use today. The disciples of Pythagoras used this theory to justify the fact that pleasant sounds were obtained by plucking a string taut at two points such that the division was made into parts in specific proportions, known today as chords. The pattern between the different musical notes was established by means of a construction that successively applied the 3:2 ratio, giving rise to the division into the notes C, D, E, F, G, A, B, in the Pythagorean scale.

The division proposed by the Pythagoreans did not completely satisfy the musical theorists of the Renaissance, because in different scales there were small dissonances in the supposed chords. In addition, there were different jumps between several notes (tones and semitones), which led the musicians to complete the scale up to 12 notes, using flats and sharps. Bach's idea was none other than to use a geometric progression of constant ratio to perform the division of the interval, the German composer obtaining as a solution the twelfth root of two as a constant of proportionality. Bach's tempered system is practically identical to that of the Pythagoreans, since the G note of both scales is almost indistinguishable to the human ear, since the seventh power of the constant of proportionality practically coincides with 3:2.

Once the pitch frequency of a note is fixed, the rest are obtained by multiplying by the proportionality constant. There are also relationships between the different frequencies and their arithmetic, geometric and harmonic averages, interpreted in the scale as the intervals of fifth, octave and fourth, respectively. A good and more extensive exposition of the subject can be found in Arenzana and Arenzana (1998).

Today we also know the physical basis governing the phenomenon of chords: the so-called octave interval, notes in the ratio 2:1, or chords in the ratio 3:2, the fifth, produce pleasant sounds because the auditory signals generated are represented by two waves whose lengths (equivalently, frequencies) make their crests coincide regularly and systematically, as exemplified in Figure 2.

Figure 2. Representation of two simple waves, on the left in 2:1 ratio and on the right in 3:2 ratio.



It should be noted that this relationship between Mathematics and Music is even more profuse and led figures such as Architas of Tarentum or Plato himself to take an interest in the study of music as part of the formation of the educated man. Subsequently, the incipient medieval schools would organize their curricula with the division of disciplines into the Trivium and the Quadrivium. The first group included Grammar, Rhetoric and Dialectic, while the second group included Arithmetic, Geometry, Music, and Astronomy. From the vision of the 21st century, it is surprising to see Music as a scientific discipline, but the truth is that this classification was based on a Platonic duality between the world of ideas and the sensible perception when understood in its static or dynamic state. In this classification, music is conceived as arithmetic in motion, which reminds us of the phrase attributed to Leibniz that "music is the pleasure of the mind when it counts without realizing what it is telling".

Figure 3. *The four disciplines of the Latin Quadrivium.*

	At rest	In movement
Number	Arithmetic	Music
Magnitude	Geometry	Astronomy

4.2. GEOMETRY AND ART

As in music, the development of art, and particularly of painting, had its eclosion with the mathematical advances that eventually configured a very productive branch of this theoretical discipline.

Since the beginnings of human civilization, Man has endeavored to make representations of reality. Thus, we have the rudimentary examples of cave

drawings, very schematic, the Egyptian figures characterized by their hieratic posture in implausible positions, and in general an infinity of representations of daily or religious scenes during the Middle Ages that do not keep proportions between objects or characters in different planes of a scene. However, Renaissance painters had already learned to give depth to their paintings and coherence between the arrangement of the elements of the painting is observed.

The work and theoretical studies of Renaissance artists such as Albrecht Dürer, Leonardo Da Vinci and Raphael Sanzio made it possible to represent the three-dimensional world on a flat canvas. Progress was based on a combination of technological, mathematical, and pictorial knowledge. From artifacts such as camera obscuras, painters were able to frame a scene and make reliable representations of reality in which planes and imaginary vanishing lines could be identified, enclosing the scene, and giving the sensation of depth. In particular, the parallelism between straight lines of reality was transformed into straight lines whose intersection in the painting was produced on the horizon lines. These results were recovered during the 17th century, formulating Theorems such as those of Desargues or Pascal, the first key results of the branch that would come to be known as Projective Geometry.

It is true that, until the emergence of Projective Geometry, mathematicians in particular and scientists and artists in general had been negatively influenced and limited by the axioms of Euclidean Geometry. In the Greek mathematical work par excellence, *The Elements*, the mathematician Euclid had organized his compendium on the basis of five axioms. The fifth of these, known as the postulate of parallels, was the subject of controversy from Euclid himself until the 19th century.

We can enunciate this postulate not in the convoluted way Euclid did in the third century B.C. but by means of another equivalent statement: given a straight

line and a point outside it, there is only one line parallel to the given line and passing through the given point. Well, the foundations of Projective Geometry revealed that the mathematical world was faced with a model of Geometry, arising from the studies of an artistic discipline, which was perfectly consistent, but contradicted Euclid's Fifth Postulate.

This new geometry opened the door to a whole new exotic world, that of non-Euclidean Geometries, whose existence seemed to be dismissed out of hand even by the most eminent figures in the discipline. Lobachevski and Bolyai proved the independence of the Fifth Postulate and figures such as Gauss, Klein, Poincaré or Riemman were in charge of constructing different non-Euclidean models, such as the sphere with its maximal circles or the hyperbolic plane. The way of measuring distances in these contexts is different from the usual one through straight lines, but it keeps certain analogies. And it would be the work of Noëther or Minkowski, at the beginning of the twentieth century, in relation to non-Euclidean metrics, the mathematical basis that allowed Einstein to formulate the Theory of Relativity.

4.3. INFINITESIMAL CALCULUS AND PHYSICS

Perhaps the most typical and fruitful relationship between two disciplines throughout history has been that between Mathematics and Physics. From the most basic studies of physical phenomena such as the fall of bodies or rectilinear and circular displacements, to more complex questions such as the determination of planetary trajectories or the form of heat diffusion, among others, a multitude of problems have stimulated the development of very powerful mathematical tools that, in addition to representing a theoretical advance, have also provided answers to the questions posed by Physics. Moreover, the branches of Mathematics have subsequently advanced independently of Physics, until, years later, the latter has found an interpretation

for purely theoretical concepts that were already developed.

It is evident that physical situations were the engine of thought to polish the idea of infinitesimal process. Euclid already resorted to an infinitesimal division of the circle to try to justify his formula for area. Democritus of Abdera and Eudoxus of Cnidus also considered the calculation of volume by means of an infinitesimal reasoning which, in fact, was possibly very similar to that described by the Italian physicist Cavalieri in the 17th century, and which gives its name to his well-known principle for the calculation of volumes of two bodies by comparison. In this same century Kepler's reasoning appears, which replicates Archimedes' results, and Galileo begins to value the infinitely large and the infinitely small.

The origins of infinitesimal calculus can also be traced back to the Middle Ages. The revision of the Aristotelian theory of momentum led Nicolas d'Oresme, one of the main exponents of the scholastic tradition in the 14th century, to conclude that, for any uniformly accelerated rectilinear motion, there exists another motion of constant velocity, such that the distance traveled in an interval is the same for both motions. We have used terminology typical of physics, but in the mathematical sense it is no more than a mean value principle for linear functions.

Back in the 17th century, one can already sense in the works of Fermat and Barrow that the notion of derivative and integral as we know it is about to blossom. Finally, in the 18th century, it would be Newton and Leibniz who would go down in the annals of history sharing, and disputing, the honor of being the fathers of infinitesimal calculus. Differential calculus was interpreted as a method of tracing tangents to curves, and therefore had to do with displacements, velocities and accelerations, while integral calculus was devoted to solving quadratures, i.e., calculations of areas under curves. It was astonishing that these

two problems were, in fact, one the inverse of the other as Cauchy demonstrated in the 19th century.

From the discovery of this powerful tool in the 17th century until the full rigorization of infinitesimal calculus at the end of the 19th century, two centuries of dizzying scientific production passed in which mathematicians advanced in theoretical results while physicists found an ideal corset to describe problems that until then had been resisted: well-known results such as L'Hôpital's rule or Rolle's and Bolzano's theorems are published, D'Alembert formalizes the idea of limit and gives a solution to the problem of the vibrating string as well as Euler and D. Bernoulli, Fourier gave a solution to the heat diffusion problem in the form of a trigonometric series, Dedekind, Weierstrass and Riemann gave examples of counter-intuitive functions... and in short, in little more than a hundred years some problems that both Physics and Mathematics had been studying for centuries were solved.

4.4. CHANCE AND ECONOMY

We end this brief sampler by talking about chance experiments, studied in Mathematics by the branch of Probability and Statistics, and their interaction with some aspects of Economics.

Games of chance may be as old as the first civilizations. They probably arose as pure amusement but found an incentive in betting systems with the emergence of money. Within this framework, that of a game of chance with monetary bets, in the 17th century the Chevalier de la Mere asked Pascal, by correspondence through Father Mersenne, what should be the distribution of a bet amount if the game had to be interrupted. The resolution given by Pascal and Fermat to this problem is considered as the beginning of Probability Theory.

In Pascal's *Pensées* one can glimpse a tendency towards the probabilistic mode of reasoning. Pascal gives an argumentation of the existence of God in terms of presupposing such existence or not and whether we choose to believe or not, appreciating in his discourse the recent ideas of probability. Leaving aside metaphysical reasoning, the approach has a certain analogy with the prisoner's dilemma, which exemplifies the prevalence of the common good over the individual good, a principle enunciated by John Nash, winner of the Nobel Prize in Economics in 1994.

The branch of mathematics that studies the behavior of a group of individuals in terms of rewards or penalties is called Game Theory and currently has great influence in other sciences such as psychology or sociology. Initially its application was limited to the economic field and, for example, it provides a way of explaining the somewhat unpredictable functioning of the stock market.

Chance also serves as a tool for the States to obtain revenue through lottery systems. This type of games are supposed to be carefully designed to maintain the interest of the bettors on the one hand, receiving sporadic small rewards in the form of minor prizes, and on the other hand, to show that there is always the possibility of a great prize, but making all this increase progressively the organizer's coffers.

From time to time there is news about citizens who have won several times a big prize. Perhaps one of the best known anecdotes in this area is that of Voltaire who, in the 18th century, took advantage of a blatant design error in the French state lottery system to amass a large fortune. In reality, all Voltaire did was follow the advice of De la Condomine, who detected an imbalance in the design of the game that led to a huge increase in the odds of winning the prize when many low-cost shares were purchased. Of course, when state lawyers and economists detected the scheme, the game was cancelled. With the fortune

amassed, De la Condomine financed an expedition to America to measure the longitude of the Earth's meridian arc.

For obvious reasons of space, we leave out of this meager list of examples many others related to Chemistry, Biology, Social Sciences, etc. The interested reader can refer to Vázquez (2001) for more examples in Science and Technology, or to Peña (2006) for more examples in Social Sciences. In any case, we have amply illustrated that the almost constant interaction of Mathematics with other fields of Human knowledge has given rise to ideas or raised questions whose answers have led to the advancement of many and diverse disciplines.

5. INTERDISCIPLINARITY IN TEACHING FROM MATHEMATICS

As we have already mentioned, both the examples in the previous section and any others chosen appropriately are perfectly valid for use in the development of the mathematics curriculum. In addition, we have legislation that can bring about a change in the methodological paradigm in the sense of interdisciplinarity, and teachers, together with the rest of the educational agents, must promote effective actions to implement this change.

It is undeniable that in today's classrooms we have a very heterogeneous student body, both in their interests and motivations and in their abilities, levels of education and family environment (Valdés et al., 2021), but we must try to instill in all of them something of the way of doing and thinking about mathematics, which has been called mathematical competence in its different and broader meanings.

Mathematical competence can be defined as the ability to understand, judge, do and use mathematics in a variety of contexts and situations in which mathematics plays or can play a role (Niss, 2002) and precisely the fact that mathematics is present in a multitude of real-life situations makes it especially

recommendable as a germ of interdisciplinary curriculum development. In fact, this is how the new LOMLOE curriculum seems to have been designed and the so-called learning situations are an opportunity for this (Gil Blanco et al., 2022). Moreover, we add, a possible good starting point is the search for and design of learning situations and interdisciplinary activities (Búa, 2020) based on the descriptors of all other subjects, which can and should influence the acquisition of Mathematical and STEM Competence (Búa, 2020).

Alsina and Mulá (2022) also point out the contributions of the National Council Teachers of Mathematics to bring together the Mathematics curriculum from a dozen standards, as well as the conceptualization of Mathematical Competence, according to the OECD, as the ability of an individual to identify and understand the role that mathematics has in the world, make informed judgments and use and engage with mathematics when needs arise for their individual life as a constructive, engaged and reflective citizen. In fact, Alsina (2017) already proposed a model of Mathematical Literacy in Childhood that, with due adjustments, can be valid in adolescence. The six phases of the model are: mathematization of the context, prior knowledge, contextualized work, co-construction and reconstruction, formalization of learning and, finally, systematic reflection.

To this model we can add the now classic decalogue promulgated by Professor Puig-Adam (1960), which seems to have been made only a few years ago and with the current educational paradigm in mind:

- 1.- Not to adopt a rigid didactic, but to mold it in each case to the student, constantly observing him.
 - 2.- Not to forget the concrete origin of mathematics or the historical processes of its evolution.
 - 3.- To present mathematics as a unit in relation to natural and social life.
 - 4.- To carefully keep the abstraction planes.
-

- 5.- To teach by guiding the creative and discovering activity of the student. To stimulate such activity by awakening direct and functional interest in the object of knowledge.
- 7.- To promote, as much as possible, self-correction.
- 8.- To achieve a certain mastery in the solutions before automating them.
- 9.- To take care that the expression of the student is a faithful translation of his thought.
- 10.- To procure to every student successes that avoid his discouragement.

In the last two decades there have appeared several descriptions of didactic proposals of interaction of Mathematics with other Sciences (Íñiguez Porras, 2015), Music (López and Gustems, 2007; Casals et al., 2014) or even Physical Education (Nieto-Isidro and Moro Domínguez, 2020). We highlight the manual of the pioneer Grup Vilatzara (2006), which has a motley collection of contextualized activities that can easily attract other disciplines to evolve into interdisciplinary proposals with the corresponding adaptations.

From the examples we have described, we can organize interdisciplinary proposals that replicate historical facts or can serve as motivation to raise questions that, as they are solved, lead to controversies. Thus, from the Mathematics Department and with a possible fit in different courses, we can propose designs that involve Music to work on proportionality, Plastic Arts and Drawing to work on perspective, Physics and Chemistry to deal with the displacement of mobiles or gravity, Biology and Geology or even Geography and History to deal with issues of the Earth, Economics for probabilistic and statistical issues and, of course, History and Philosophy can always play a role that provides a sense of background to the moment in which the relationships, problems and their solutions arose.

~~Of course, each of the proposals in the previous paragraph is worthy of a~~

detailed study for its development and application in the classroom. However, since this is not the main objective of this article, but rather the presentation of an outline of possibilities in the line of interdisciplinary action, we encourage teachers to implement and share their proposals for their students in this regard.

6. CONCLUSIONS

In the 21st century, we find ourselves in a context of globalization in which the competence of the individual prevails over the agglomeration of simple knowledge, surrounding us with a complex reality in which each phenomenon can be observed from different points of view. Each discipline can contribute its own knowledge, but the integration of these is indispensable to understand the whole phenomenon and generate a higher level of knowledge that would otherwise be impossible.

On the other hand, despite the shortcomings detected, the educational system in Spain seems to be already predisposed to introduce interdisciplinarity in the classroom as a facilitating approach to the teaching-learning process for the acquisition of the competences identified by supranational organizations and reflected in legislation.

Given the ample evidence of the interaction of Mathematics with many other areas of knowledge, and the fact that mathematical competence has been identified as one of those that the individual should develop in his or her education, Mathematics stands as a fundamental vertex around which to articulate interdisciplinary proposals.

With the conscientious work of teachers, in collaboration with the Administration and other educational agents, it will be possible to develop proposals that show Mathematics as useful in the classroom as it is in the reality

that surrounds us. We are confident that in this way we will be able to instill in students the values and skills of the discipline, which are so useful and necessary to form competent citizens with critical thinking.

REFERENCES

- Alsina, Á. (2017). Caracterización de un modelo para fomentar la alfabetización matemática en la infancia: vinculando la investigación con buenas prácticas. *Avances de Investigación en Educación Matemática*, 12, 59–78.
- Alsina, Á & Mulà, I (2022). Sumando competencias matemáticas y de sostenibilidad. Implementar y evaluar actividades interdisciplinares. *UNO: Revista de Didáctica de las Matemáticas*, 95, 23–30.
- Arenzana, V. & Arenzana, J. (1998). Aproximación matemática a la música. *Revista Números, de didáctica de las matemáticas*, 35, 17–31.
- Boyer, C. B. (1986). *Historia de la Matemática* (Trad, M. Martínez). Alianza (original en inglés publicado en 1968).
- Búa, J. B. (2020). Implementación de actividades de modelización, STEM y Maker en Enseñanza Secundaria. *Números: revista de didáctica de las matemáticas*, 104, 82–102.
- Campos, I., & Zúñiga, J. Á. (2020). Composición de los claustros: ¿cómo afecta al desempeño académico de los centros de Educación Secundaria? *Revista de educación*, 387, 265–289. DOI.10.4438/1988-592X-RE-2020-387-435.
- Casals, A., Carrillo, C. & González-Martín, C. (2014). La música también

cuenta: combinando matemáticas y música en el aula. *Revista LEEME Lista Electrónica Europea Música en la Educación*, 34, 1-17.

- Consejo Escolar del Estado (2022). *Informe sobre el estado del Sistema Educativo. Curso 2020-2021*. Secretaría General Técnica, Centro de Publicaciones del Ministerio de Educación y Formación Profesional.
 - Delors, J. (1996). *La educación encierra un tesoro. Informe a la UNESCO de la Comisión Internacional sobre la Educación para el siglo XXI*. UNESCO.
 - Estefanía Lera, J.L. (2017). La Inspección ante la innovación educativa. *Avances en Supervisión Educativa*, 27. <https://doi.org/10.23824/ase.v0i27.591>
 - Fiallo, J. (2001). La interdisciplinariedad en la escuela: Un reto para la calidad de la educación. *Pueblo y Educación*.
 - Follari R. (2007). La interdisciplina en la docencia. *Polis*, 16. <https://journals.openedition.org/polis/4586>
 - García Pérez, F. F. & de Alba Fernández, N. (2008). ¿Puede la escuela del siglo XXI educar a los ciudadanos y ciudadanas del siglo XXI? *Scripta Nova: Revista electrónica de Geografía y Ciencias Sociales*, 12 (270).
 - Gil Blanco, M. A., Martín Escanilla. R. & Muñoz Casado. J. L. (2023). Competencias específicas de matemáticas en la LOMLOE. Un cambio en el enfoque de la enseñanza y el aprendizaje en matemáticas. *Supervisión21*, 67. <https://doi.org/10.52149/Sp21/67.9>
-

- Gómez, I. (2020). Visión social del profesorado de Educación Secundaria Obligatoria. *Quaderns digitals: Revista de Nuevas Tecnologías y Sociedad*, 91.
- Gozzer, G. (1982). Un concepto aún mal definido: la interdisciplinariedad. *Revista trimestral de Educación*, UNESCO, XII (3), 301–314.
- Grup Vilatzara (2006) *¿Es posible viajar con las Matemáticas?* Federación Española de Sociedades de Profesores de Matemáticas, Institut de Ciències de l'Educació de la Universitat Autònoma de Barcelona.
- Guzmán, M. de. (2007). Enseñanza de las Ciencias y la Matemática. *Revista Iberoamericana de Educación*, 43, 19–58.
- Íñiguez Porras, F. J. (2015). El desarrollo de la competencia matemática en el aula de ciencias experimentales. *Revista Iberoamericana De Educación*, 67 (2), 117–130. <https://doi.org/10.35362/rie672256>
- Kline, M. (1976). *El fracaso de la matemática moderna* (Trad. S. Garma). Siglo XXI (original en inglés publicado en 1973).
- Ley Orgánica 3/2020, de 29 de diciembre, por la que se modifica la Ley Orgánica 2/2006, de 3 de mayo, de Educación [LOMLOE] <https://www.boe.es/eli/es/lo/2020/12/29/3>
- Llano Arana, L., Gutiérrez Escobar, M., Stable Rodríguez, A., Núñez Martínez, M., Masó Rivero, R. & Rojas Rivero, B. (2016). La interdisciplinariedad: una necesidad contemporánea para favorecer

el proceso de enseñanza aprendizaje. *MediSur*, 14 (3), 320–327.

- López, P. & Gustems, J. (2007). Reflexiones y dificultades interdisciplinares: una experiencia conjunta de matemáticas y música. *UNO: Revista de didáctica de las matemáticas*, 44, 110–116.
- López, L., (2012). La importancia de la interdisciplinariedad en la construcción del conocimiento desde la filosofía de la educación. *Sophia, Colección de Filosofía de la Educación*, (13), 367–377.
- López Beltrán, M., Albarracín Gordo, LL., Ferrando Palomares, I., Montejo Gámez, J., Ramos Alonso, P., Serradó Bayés, A., Thibaut Tadeo, E. & Mallavibarrena, R. (2020). La educación matemática en las enseñanzas obligatorias y el bachillerato. En Martín de Diego, D.(coord.) (2020). *Libro blanco de las Matemáticas* (pp. 1–94). Fundación Ramón Areces, Real Sociedad Matemática Española.
- Organización para la Cooperación y el Desarrollo Económicos [OCDE]. (2021). Marco de Aprendizaje 2030 – Brújula 2030. <https://bit.ly/31NUtmy>
- Merchán Iglesias, F. J. (2021). La política educativa de la democracia en España (1978- 2019): Escolarización, conflicto Iglesia–Estado y calidad de la educación. *Archivos Analíticos de Políticas Educativas*, 29 (61). <https://doi.org/10.14507/epaa.29.5736>
- Nieto–Isidro, S. & Moro Domínguez, M. A. (2020). Refuerzo interdisciplinar de las combinaciones numéricas básicas en Educación Primaria. *Educación Matemática*, 32 (3), 153–177. <https://doi.org/10.24844/em3203.06>

- Niss, M. (2002). *Mathematical competencies and the learning of mathematics: the Danish Kom Project*. Roskilde University.
- Novella, C., & Cloquell, A. (2022). Falta de consenso e inestabilidad educativa en España. *Revista complutense de educación*, 33 (3), 521-529. <https://doi.org/10.5209/rced.74525>.
- Peña, D. (2006). Las Matemáticas en las Ciencias Sociales. *Encuentros multidisciplinares*, 8 (23), 67-79.
- Pérez-Díaz, V. & Rodríguez, J. C. (2013). *Educación y prestigio docente en España: la visión de la Sociedad*. En *El prestigio de la profesión docente en España: visión y realidad* (pp. 33-108). Fundación Europea Sociedad y Educación, Fundación Botín.
- Piaget, J. (1978): *Introducción a la epistemología genética*. Paidós.
- Puig-Adam, P. (1960). *La Matemática y su enseñanza actual*. Centro de publicaciones del Ministerio de Educación Nacional.
- Pozuelos Estrada, F., Rodríguez Miranda, F. & Travé González, G. (2012). El enfoque interdisciplinar en la Enseñanza universitaria y el aprendizaje basado en la investigación. Un estudio de caso en el marco de la formación. *Revista de Educación*, 357, 561-585.
- Quirós, A. & Vázquez, J. L. (2006). *Las matemáticas como fuerza interdisciplinar*. *Encuentros multidisciplinares*, 8 (23), 3-4.
- Sáenz Martínez A. (2022). El asesoramiento técnico-educativo de la inspección como factor de mejora de la calidad de la Educación. *Supervisión*21, 63. <https://doi.org/10.52149/Sp21>

- Sahle–Work Zewde, S. E. (coord.) (2021) *Reimaginar juntos nuestros futuros: un nuevo contrato social para la educación. Informe de la Comisión Internacional sobre los futuros de la Educación*. UNESCO.
- Torres J. (1994) *Globalización e Interdisciplinariedad: el curriculum integrado*. Morata.
- Valdés Durán, Y. (2017). Integración curricular: un concepto que tensiona a las disciplinas escolares. *Comunicaciones en Humanidades*, 3, 146–154.
- Valdés, M., Sancho, M. A. & de Esteban, M. (2021). *Indicadores comentados sobre el estado del Sistema Educativo Español*. Fundación Ramón Areces, Fundación Europea Sociedad y Educación.
- Vázquez, J. L. (2001). *The importance of Mathematics in the development of Science and Technology*. Boletín de la Sociedad Española de Matemática Aplicada, 19, 69–112.
- Vázquez Cano, E. (2018). La participación de la Inspección Educativa para el asesoramiento y la supervisión de la innovación escolar. *International Studies on Law and Education* 29/30. 179–194.
- Welch, J. (2011). El nacimiento de la interdisciplinariedad a partir del pensamiento epistemológico. *Issues in integrative studies*, 29, 1–39.
- Wigner, E. (1960). The unreasonable effectiveness of Mathematics in the Natural Sciences. *Communications in Pure and Applied Mathematics*, 13 (1), 1–14.